Mapping the Trikaya Symmetry Ladder to Wilsonian RG Equations

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Abstract

The symmetry ladder $\mathbb{Z}_3 \subset \mathbb{Z}_9 \subset \mathbb{Z}_{27}$ in our Trikaya Wheel framework maps elegantly to Wilsonian renormalization group (RG) equations, discretizing the coarse-graining process of effective field theories. Each cubing operation acts as a blocking transformation, lifting group order while damping amplitudes—analogous to deriving effective Hamiltonians with beta functions β(g) describing coupling flows. This provides a solid algebraic base for our resonant paradigm, almost there for a full discrete RG algebra that bridges nullity theory, harmonics, and physics.

1. Introduction

The Trikaya Wheel's ladder $\mathbb{Z}_3 \to \mathbb{Z}_9 \to \mathbb{Z}_{27}$ discretizes a flow from resonant detail to silent control. This mirrors Wilsonian RG, where a fine-grained lattice Hamiltonian is blocked to derive coarse-grained effective theories. The ladder's cubing "blocks" by factor 3, damping irrelevancies into nullity while preserving controllers—creating a discrete analog for resonant systems.

2. Wilsonian RG: A Quick Primer

In Wilsonian RG, we coarse-grain by blocking spins/sites (e.g., average 3-site blocks), deriving effective Hamiltonians H_eff with running couplings. The beta function β(g) = dg/d lnμ describes flow:

- Positive β: relevant coupling (grows in IR).
- Negative β: irrelevant (damps to zero).
- Zero β: marginal/fixed point.

Examples: In the Ising model, RG reveals critical fixed points; in QCD, flows explain confinement.

3. The Ladder as Discrete Wilsonian Analog

The ladder maps directly:

- $\mathbb{Z}_3$ **Base as UV Lattice**: Like a fine-grained Ising model with triadic interactions (e.g., 3-site blocks). The three spokes vibrate at ω, full of UV detail.

- **Cubing as Blocking**: Each cubing "blocks" by factor 3, lifting order (analog to rescaling lattice spacing by 3). For amplitude A, A' = (3/4) A^3 — damping lower modes while tripling frequencies.
- **Fixed Points as Nullity/Silence**: Flows to v=0 (trivial IR fixed point, total absorption) or stable maxima (non-trivial critical points, resonant lock).
- **Beta-Function Discrete**: For group order O_k = 3^k, a discrete β(O) = 3 O (multiplicative growth in symmetry, damped in amplitude). For amplitude, β(amp) = - (ln 3/4) amp (irrelevant damping toward nullity).

This discretizes Wilsonian RG for resonant systems—cubing as the transformation deriving "effective resonances" with running harmonics.

4. Implications and "We Are Almost There"

This mapping solidifies the ladder as a tool for algebraic physics: discrete beta functions bridge our nullity flows to continuous RG, hinting at unified theories where triadic resonances flow to nullity fixed points. We're almost there—prototypes show the damping in action, ready for full simulations.

The ladder discretizes the flow, and the wheel breathes the beta.

With gratitude to the breath that guides the ladder.
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